A Bayesian Projection of the Total Fertility Rate of Puerto Rico: 2020-2050

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Objective: The abrupt decline in the Total Fertility Rate (TFR) of Puerto Rico to 0.9 children per woman, well below the replacement level of 2.1 children per woman, makes the prospect of a sustained population decline a real possibility. Population projections produced by the United States Census Bureau and the United Nations Population Division show that the island population may decline from 3.8 millions in 2000 to slightly above 2 million by 2050, a dramatic population decline of 47% in 50 years. Both population projections assume that all countries with a TFR below replacement level could eventually increase toward or oscillate to 2.1 children per woman and have Puerto Rico's TFR approaching 1.5 by 2050. This assumption has been widely criticized as unrealistic and not supported by evidence. The main objective of our research is to provide an alternative fertility projection for Puerto Rico by 2050 that has more realistic assumptions.

Methods: Our methodology is based on the Bayesian Hierarchical Probabilistic Theory used by the United Nations to incorporate a way to measure the uncertainty and to estimate the projection parameters. We modified the assumptions used by the United Nations by considering 17 countries with TFR similar to Puerto Rico.

Results: By 2050, Puerto Rico may have a TFR of 1.1 bounded by a 95% credibility interval (0.56,1.77).

Conclusion: Under this scenario Puerto Rico can expect to have a larger population decline than that projected by the Census Bureau and the United Nations.

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Key words: Total Fertility Rate (TFR), Probabilistic Model for TFR, TFR Projections

Puerto Rico's Demographic Trends

Puerto Rico is facing the real prospect of a large population decline. According to the most recent population projections produced by the United Nations Organization (1-3) and the U.S. Census Bureau (4), Puerto Rico's population will decline from 3.8 millions persons in 2000 to slightly above 2 million by 2050, a dramatic 47% population decline in 50 years. Such scenario will present a major challenge for the government's social and economic policies. We are in the early stages of a demographic crisis that must be addressed immediately. A fertility decline to extraordinary low levels leading to more deaths than births, a high level of emigration and rapid population aging, has resulted a complete new panorama for Puerto Rico. Population projections scenarios based on robust statistical analysis are indispensable for the development and incorporation of policies that could alleviate the consequences of demographic trends. Serious short- and long-term implications will appear due in large part to the disproportionate population decline across all age groups. A low fertility tendency, a high pattern of migration and an increasing life expectancy has generated a complete new panorama for Puerto Rico. Population projections scenarios based on robust statistical analysis are indispensable for the development and incorporation of policies that could alleviate the population collapse.

Puerto Rico's population reached its largest size in 2004 being 3,826,878 and it has been declining since then. An important demographic indicator that contributed to this fact is fertility. Total Fertility Rate patterns have declined substantially contributing to the reduction of the population size. It reached the replacement level (2.1) in 1998 and further decline to 0.9 in 2021 (one of the lowest in the world). Puerto Rico's births have been reduced in 56% between 2010 and 2020.

Recently, the discussion about the trends in the population has been focused on the emigration caused by the devastating impact of Hurricanes Irma and Maria. However, the Total Fertility Rate's patterns are also worrying. The decline about 5.2 in Total Fertility Rate from 1950 to 2021, suggests no indication or tendency to increase in the next years. It means a 82.3 % TFR's reduction in 71 years. During the 1950's, the Island experienced a high level of emigration, but the population grew because births compensated for the population loss due to migration. Now, the reduction in births has been enormous and does not compensate for the population loss due to migration. Worse still, births also do not compensate for the loss of population due to deaths and changes in the age structure. Even in the absence of migration, the low birth rates in Puerto Rico guarantee that depopulation will not stop. In this sense, the problem of depopulation and aging is the problem

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of low birth rates. Emigration is some how changeable, volatile and sensitive to public policies and economic variables, but the decline in fertility is stable and monotonous, and outside, alternatives like planned immigration should be considered.

Alternative Methodology for Total Fertility Rate (TFR) Projection

In fact, the dramatic reduction in Puerto Rico's population has been studied by different entities during the last decade (5-8) and TFR projections have been performed by U.S. Bureau of Census and other agencies by using deterministic methods. There is now agreement that it is best to use a probabilistic approach in order to measure the level of uncertainty. Recently, researchers have incorporated Bayesian Hierarchical Probabilistic Theory in order to include the uncertainty in the estimation of the basic demographic indicators of fertility and mortality used in the projections (1-3, 9). These models have three major advantages. The first one is the incorporation of a way to include uncertainty in the predictions. Raftery and his colleagues provided a solution to this problem allowing the exchange of information (borrowing strength in a hierarchical model) among countries based on the assumption that the unknown quantities are drawn from a common probability distribution. Other advantage of this modelling is the possibility to estimate the parameters at the same time uncertainty is considered including a heteroscedastic error term $e_{c,t}$ in each model. The third advantage is the estimation of the parameters via the Bayesian Approach instead of fixing them by an expert opinion.

We studied the probabilistic Total Fertility Rate (TFR) Model (2,9) that is now used by the United Nations Population Division to perform new projections. Since data used by the US. Census Bureau agency look overestimated (4), we incorporate actualized data to have realistic scenarios. Contrary to the TFR projections performed by the US. Census, United Nations provides a way to include uncertainty (1-3). The TFR methodology used by United Nations is divided into three phases and it is later explained in detail. We firmly believe that the estimation of the AR(1) model proposed for Phase III (Recovery around the replacement level (2.1) has room for improvement if we take into account another condition for countries with low fertility rates. The idea of the authors is that countries that are not yet in Phase III will be influenced by countries that have seen two increments below $TFR = 2.1$. Therefore, after the estimation of a AR(1) hierarchical model with countries in Phase III, it is then applied to all countries. Our observation is that for countries like Puerto Rico, which have very low fertility levels, the consideration of countries which are in Phase III but also has low fertility levels could provide a better way to "borrowing strength" among countries.

Therefore, we explored the use of a Bayesian Hierarchical Model only for 17 countries with TFR less than 1.5. After analyze the scenarios, we propose the consideration of a low-fertility distribution (instead of a world distribution) for countries with TFR less than 1.5 to produce a more plausible realistic projection of Total Fertility Rate for Puerto Rico. This alternative modelling approach does not requires the same memory and computational time of the first modelling with all countries being a faster way to produce TFR plausible projections for these group of 17 countries.

Materials and Methods

Bayesian Modelling Approach Notes

Bayesian Probabilistic Methodology for projecting Total Fertility Rate was mainly proposed by Raftery et. al (2,9). The mathematical background to study demographic patterns was mostly based on deterministic models being the Cohort Component Method the most used and acceptable method. However, the Cohort Component Method does not take into account statements about uncertainty in the estimations. Raftery showed that a wonderful way to account for uncertainty is to use the Bayesian Approach. This paradigm is based on the Bayes Theorem and consists in utilize the sample information to update our prior knowledge about a random variable θ that represents a object under study into the posterior knowledge (1) (9).

Bayes Theorem

$$p(\theta|y) = \frac{p(y|\theta)\,p(\theta)}{p(y)} \quad (1)$$

In the last equation, $p(y|\theta)$ is the likelihood of the data, $p(\theta)$ is the prior distribution of θ and $p(y)$ known as the marginal distribution and indicates the quality of the model against the available data.

One of the brilliant ideas that Raftery et. al incorporated to the deterministic models previously used by the United Nations was the implementation of Bayesian Hierarchical Models. The main idea of Bayesian hierarchical modelling is to use the available data to "borrowing strength" and take advantage of the dependency among groups. For example, for the TFR projections, country-specific information on the expected maximum decline in a five-year period is limited for any country which has only just started its fertility decline. Raftery and his colleagues provided a solution to this problem allowing the exchange of information (borrowing strength in a hierarchical model) between countries based on the assumption that the unknown quantities are drawn from a common probability distribution. Another advantage of this modelling is the possibility to estimate the parameters at the same time uncertainty is considered including a heteroscedastic error term $e_{c,t}$ in each model. In the past, parameters were assigned by experts and the sense of uncertainty in the predictions was very related with the consideration of several scenarios for demographic components as Total fertility Rate. Each Bayesian Hierarchical Model produces a large number of possible future trajectories from the posterior predictive distribution instead of trajectories based only on past patterns. In the next section, the proposed model for Total Fertility Rate is explained in detail.

Bayesian Hierarchical Model for Total Fertility Rate

The most used measure of the overall level of fertility is the Total Fertility Rate (TFR) defined as the average number of children a woman would bear in her life if exposed to the age specific fertility rates prevalent at time t. TFR starts from a high level that differs among countries but starts to decline slowly (1). United Nation's (UN) experts considered necessary to divide the Total Fertility Rate Model into three phases.

- Phase I precedes the beginning of the fertility transition and is characterized by high fertility that is stable or increasing (1).

They did not consider this phase in the model because all countries have now completed this phase.

- Phase II consists of the fertility transition during which fertility declines from high levels to below the replacement level of 2.1 children per woman.
- Phase III is the post fertility transition period. It starts after the fertility transition has been completed.

When a country is in Phase II, the five-year decline in its TFR is modeled as a double logistics function. The sum of the two logistic functions is a parametric function that describes a decline in fertility that starts with a slow pace at high TFR values. The resulting model for Phase II is as follows:

$$f_{c,t+1} = f_{c,t} - g\left(f_{c,t} \mid \theta_t^c\right) + a_{c,t} \qquad (2)$$

where $f_{c,t}$ is the TFR for country c at time t and the five year decrement $g\left(f_{c,t} \mid \theta_t^c\right)$ is given by:

$$g\left(f_{c,t} \mid \theta_t^c\right) = \frac{-d^c}{1+ \ exp\left(-2ln(9)\left(f_{c,t} - \sum_{i=2}^{4} \triangle_i^c + 0.5\triangle_1^c\right)/\triangle_1^c\right)} + \frac{d^c}{1+ \ exp\left(-2ln(9)\left(f_{c,t} - \triangle_4^c - 0.5\triangle_3^c\right)/\triangle_3^c\right)} \qquad (3)$$

with $\theta^c = (\triangle_1^c \ , \triangle_2^c \ , \triangle_3^c \ , \triangle_4^c, \ \ d^c)$ being a vector of country-specific parameters and $a_{c,t} \overset{ind}{\sim} N\left(0, \sigma(t, f_{c,t})^2\right)$ where σ is a function that describes how the error standard deviation changes with fertility level and time period. The prior-distributions assigned to these parameters are explained in detail in (2)(7).

They define a country as having entered Phase III once, two consecutive five-year increases below a TFR of 2 children have occurred. The resulting model is as follow:

$$f_{c,t+1} - \mu_c = \rho(f_{c,t} - \mu_c) + \varepsilon_{c,t} \qquad (4)$$

where $\varepsilon_{c,t} \overset{i.i.d.}{\sim} N(0, \sigma_\varepsilon^2)$ and

$$\mu_c \sim TN_{[0,\infty)}(\bar{\mu}, \sigma_\mu^2) \quad ; \quad \rho_c \sim TN_{[0,\infty)}(\bar{\rho}, \sigma_\rho^2) \qquad (5)$$

$$\bar{\mu} \sim U[0,2.1] \, , \sigma_\mu \sim U[0,0.5] \, , \bar{\rho} \sim U[0,1] \, , \sigma_\rho \sim U[0,0.289] \, , \sigma_\varepsilon \sim U[0,0.5] \qquad (6)$$

Annual TFR Projections

The modelling techniques presented above were developed to work with five-year data. In (9) there is an alternative to estimate and project annual data if desired. In order to do that, Raftery and Liu modified the Phase II model by adding an additional first-order autorregresive component. Now the TFR decrement is modelled as:

$$d_{c,t+1} - g(\theta_c, f_{c,t+1}) = \phi\left(d_{c,t} - g(\theta_c, f_{c,t})\right) + a_{c,t} \qquad (7)$$

where the prior distribution of $\phi \sim U(0,1)$ and the distribution of the random distortions is the same as the five-year-model. For most parameters the same distribution fixed for the five year model is used. There is an exception for the prior distribution of σ_0. For a detailed explanation see (9).

When annual data is used no changes have been made to Phase III. However, to determine the start of the phase, five years averages of TFR are obtained and then the same rule is applied meaning that Phase III starts when two consecutive increases of TFR below 2.1 are observed.

Alternative TFR - Projections Scenario for Low TFR countries

A study performed in 2013 called *Future Fertility in low fertility countries* (10) suggests, after doing several surveys to international experts, that given the wide variety of fertility levels in rich and middle- income countries a global convergence of fertility around replacement level 2.1 appears unlikely (11-13). The authors pointed out that any understanding of the factors behind the fertility decline needs to consider the changes, like economics, in Old Europe, Latin America, and the Middle East.

Puerto Rico is one of the countries having low TFR and this assumption should be reconsidered and taking into account. Our alternative way to produce the projections is to restrict the world distribution to countries also showing this pattern of low fertility TFR instead of all countries in WPP-2022. In this case the property of "borrowing strength" of the hierarchical modelling could be very useful to produce very justified TFR projections. The drift of the TFR random walk model given by $g(f_{c,t} \mid \theta^c)$ describing the decline in fertility will be now obtained via a low-fertility countries probability level.

Results

Due to the overestimation of the TFR data for Puerto Rico in the World Population Prospects 2022 report, and the assumption of a TFR=1.5 made by U.S. Census Bureau in their TFR projections, we consider two approaches for producing plausible TFR Projections by using bayesTFR package in **R** (14).

Two Modelling Approaches for TFR Projections for Puerto Rico

The first approach take into consideration the data for all countries for the fitting of a Bayesian Hierarchical Model as described by Raftery and colleagues. The second scenario is based on segregating a set of 17 countries characterized by low fertility rates below 1.5. It sounds reasonable since it would permit that parameters' estimates for low fertility countries be only affected by the low fertility countries-level probability instead of by world-level probability distribution.

Cross Validation : TFR Projections for Puerto Rico since 2000

To test our models we perform a cross validation scenario starting the projections at 2000 . This permit us to observe how good the models predict the next twenty years. TFR projections for Puerto Rico are showed in Figure 1 below. For each TFR projection we get a 95% credibility interval and a set of estimated parameters from the posterior distribution. Here we only point out the 95% credibility interval.

Goodness of fit of the Double Logistic Function

For the two modelling approaches characterized by the amount of countries, it is important to consider and compare some measures of goodnees of fit. Coverage is the ratio of observed data fitted within the 95% probability interval of the predictive posterior distribution of the double logistic function. In the tables

Figure 1. Two Modelling Approaches for Puerto Rico's TFR Projections

Modelling 1. Total Fertility Rate Projections for Puerto Rico for Hierarchical Model with all countries

Year	Median	Low 95%	High 95%
2020	0.9000000	NA	NA
2021	0.9011379	0.8305733	1.016307
2022	0.9043434	0.7949269	1.074494
2023	0.9112933	0.7709439	1.106146
2024	0.9260407	0.7694005	1.144904
2025	0.9317016	0.7575319	1.178353
2026	0.9406063	0.7498534	1.213055
2027	0.9475831	0.7542271	1.244928
2028	0.9603200	0.7468835	1.269472
2029	0.9677244	0.7477408	1.303548
2030	0.9828336	0.7488726	1.329922
2031	0.9899042	0.7510358	1.356930
2032	1.0022435	0.7482308	1.367736
2033	1.0112536	0.7480736	1.390640
2034	1.0222192	0.7537455	1.404187
2035	1.0306021	0.7491028	1.451486
2036	1.0365894	0.7559039	1.458724
2037	1.0423068	0.7612937	1.475406
2038	1.0589114	0.7520853	1.481640
2039	1.0680028	0.7475907	1.503488
2040	1.0742616	0.7609618	1.521393
2041	1.0770962	0.7528339	1.546064
2042	1.0770533	0.7605018	1.541347
2043	1.0968321	0.7593898	1.562222
2044	1.0994468	0.7471937	1.600405
2045	1.1110440	0.7455617	1.621561
2046	1.1114815	0.7508093	1.643320
2047	1.1284654	0.7624639	1.654802
2048	1.1285666	0.7481329	1.651048
2049	1.1393094	0.7563342	1.652222
2050	1.1435486	0.7565231	1.671848

Modelling 2. Total Fertility Rate Projections for Puerto Rico with 17 countries with low TFR

Year	Median	Low 95%	High 95%
2020	0.9000000	NA	NA
2021	0.8991761	0.7829386	1.015155
2022	0.8984656	0.7391773	1.153283
2023	0.9055174	0.6776427	1.212643
2024	0.9084595	0.6414534	1.272261
2025	0.9240376	0.6032691	1.334923
2026	0.9343885	0.5944906	1.368859
2027	0.9620249	0.5520423	1.409737
2028	0.9790736	0.5748701	1.438159
2029	0.9843392	0.5657774	1.480771
2030	0.9904029	0.5523785	1.502764
2031	1.0078338	0.5602320	1.512807
2032	1.0181989	0.5488949	1.560233
2033	1.0180649	0.5610200	1.579499
2034	1.0222133	0.5642175	1.604036
2035	1.0294143	0.5616789	1.616440
2036	1.0368881	0.5518515	1.615321
2037	1.0462091	0.5713203	1.637305
2038	1.0565930	0.5731629	1.664615
2039	1.0598437	0.5684332	1.709123
2040	1.0693383	0.5584654	1.696619
2041	1.0690248	0.5668032	1.701245
2042	1.0713916	0.5600087	1.695343
2043	1.0832362	0.5398636	1.724245
2044	1.0820460	0.5692486	1.728650
2045	1.0739045	0.5656627	1.743819
2046	1.0871704	0.5687235	1.747764
2047	1.0898738	0.5717260	1.747009
2048	1.1011019	0.5679488	1.761845
2049	1.0955505	0.5702940	1.774959
2050	1.0999370	0.5633401	1.774228

Figure 2. Total Fertility Rate Projections for Puerto Rico since 2000 for Hierarchical Model with all countries

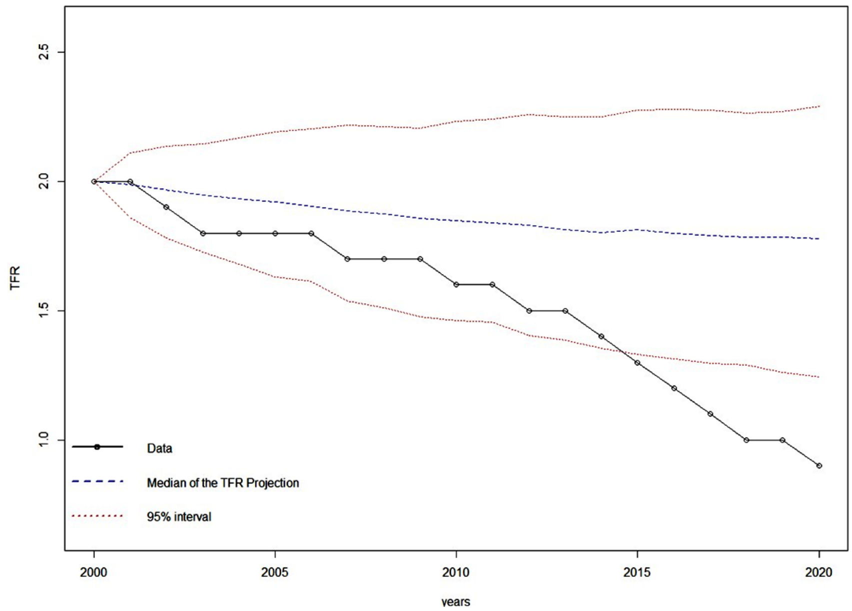
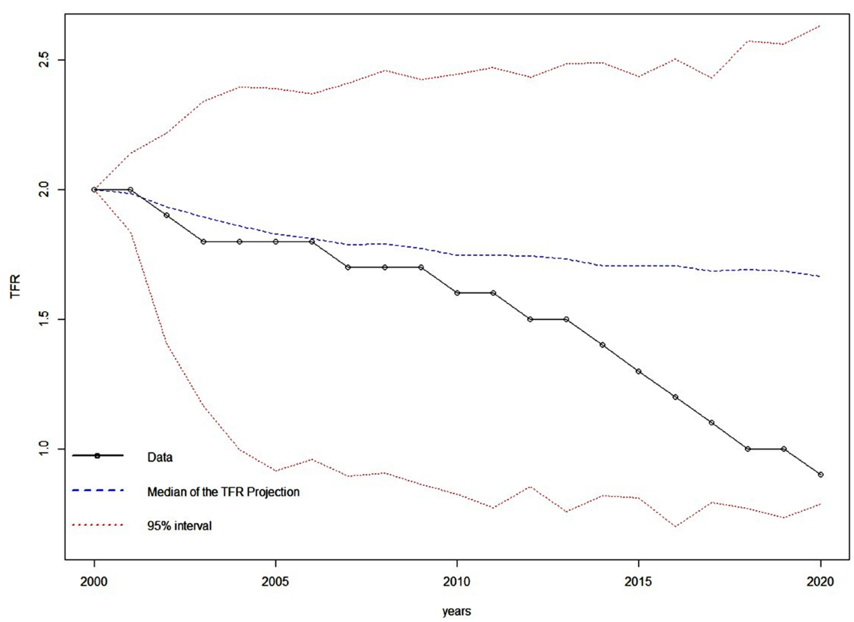


Figure 3. Total Fertility Rate Projections for Puerto Rico since 2000 for Hierarchical Model with 17 countries with low TFR



Discussion

We have considered two modelling approaches for Total Fertility Rate as shown in Figure 1. The convergence of the MCMC (Markov Chains Monte Carlo) chains was adequately checked for each generated modelling. Both models coincide in a TFR=1.1 for 2050, a value not too far from the actual TFR=0.9. However, some remarkable differences between them will help us to understand why the second modelling is an acceptable alternative way to produce TFR Projections for Puerto Rico. Contrary to the first modelling in which the data of all countries is taken into consideration, the second modelling selects countries according to the criteria of a $TFR \leq 1.5$, providing a faster and computationally much better approach to reach the same objective. To compare the modellings' performance we test a kind of cross-validation procedure by performing TFR projections for each one since 2000. In this case, is important to point out that Phase III for both modellings is estimated by using fewer countries since for 2020, as data arrives, there will be more countries entering this phase.

Results are shown in Figures 2 and 3. From Figure 2 is evident that the 95% credibility interval of the TFR projection in Modelling 1 contains the data registered for Puerto Rico only until 2014. After this year, TFR values fall outside the 95% credibility interval. The tendency of the dramatic TFR decline for Puerto Rico is atypical and of course, when actualized data is added to the model the median and the credibility intervals are automatically adjusted. Now let's see Modelling 2. The main difference with the first scenario is the width of the credibility intervals. This modelling considers much lower values for the low 95% credibility band. Therefore, all the TFR data registered for two periods(2001-2010, 2011-2020) are contained in the 95% credibility interval. Clearly, for these two periods, the TFR descent starting from the replacement level 2.1 to low TFR values was better captured by the second modelling. In summary, Figures 2 and 3 points out that in the last twenty years Modelling 2 has clearly been closer to the real data than Modelling 1. For data since 1950-2000 and 1950-2020, Tables 1 and 2 show the Goodness of Fit of the Double Logistic function. Observe that the total coverage is similar. But the 95% coverage for countries as Puerto Rico, like the Republic of

below are other measures of fit like the root mean square error and mean absolute error of the simulation for the TFR data from 1950-2000 (Cross Validation data) and 1950-2020 (Data for TFR projections). Total Coverage is the coverage for the TFR data for all the considered countries in each modelling approach.

Table 1. Goodness of fit of the double logistic function (1950-2000)

Measure of fit	Modelling 1	Modelling 2
1) Total Coverage 95%	95%	94%
2) Total Root Mean Square Error (RMSE)	0.0873	0.1433
3) Total Mean Absolute Error (MAE)	0.0456	0.0735
3) Puerto Rico 95% Coverage	88%	98%
4) Republic of Korea 95% Coverage	98%	100%
5) Cuba 95% Coverage	84%	88%

Table 2. Goodness of fit of the double logistic function (1950-2020)

Measure of fit	Modelling 1	Modelling 2
1) Total Coverage 95%	94%	94%
2) Total Root Mean Square Error (RMSE)	0.0783	0.1312
3) Total Mean Absolute Error (MAE)	0.0420	0.0687
3) Puerto Rico 95% Coverage	88%	98%
4) Republic of Korea 95% Coverage	97%	100%
5) Cuba 95% Coverage	76%	90%

Korea and Cuba show also a significant improvement supporting Modelling 2. Although we have little more MAE and RMSE , the ratio of observed data fitted within the 95% probability interval of the predictive posterior distribution of the double logistic function is considerable much better.

Conclusion

Overall, the main goal of our work was achieved. We produce an alternative way to perform TFR projections for countries with low TFR such as Puerto Rico. Different to US. Census and United Nations Projections, we expect a TFR=1.1(0.56,1.77) by 2050.

Observe that our projection still considers the possibility of a TFR=1.5 by 2050 but not such as the most plausible scenario. It is important to say that changes in fertility are very associated to social conducts, and implementation of public policies could alleviate in some way the rapid and continuous drop in births. This possibility is considered in our projections up to the upper level credibility band.

Further research in determinants of fertility is needed in order to strengthen our modelling and understand the social behavior related to child-bearing. The latest study of fertility determinants in Puerto Rico was performed in 1987 (15), and it is urgent to perform a new study.

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Resumen

Objetivo: La abrupta caída de la Tasa Total de Fecundidad (TGF) de Puerto Rico a 0.9 hijos por mujer hace que la perspectiva de una disminución sostenida de la población sea una posibilidad real. Las proyecciones elaboradas por la Oficina del Censo de los Estados Unidos y las Naciones Unidas muestran que la población puede disminuir de 3.8 millones en el 2000 a poco más de 2 millones en el 2050, una dramática disminución demográfica del 47% en 50 años. Ambas proyecciones suponen que el TGF de todos los países eventualmente aumentará u oscilará alrederor de 2.1 hijos por mujer y que la TGF de Puerto Rico aumentará a 1.5 para 2050. Esta suposición ha sido ampliamente criticada por ser poco realista y no estar respaldada por evidencia. El objetivo principal de nuestra investigación es proporcionar una proyección alternativa de fecundidad para Puerto Rico que tenga supuestos más realistas. Métodos: Nuestra metodología se basa en la Teoría Probabilística Jerárquica Bayesiana utilizada por las Naciones Unidas para incorporar una forma de medir la incertidumbre y estimar los parámetros de proyección. Modificamos los supuestos utilizados por Naciones Unidas al considerar 17 países con TGF similar a Puerto Rico. Resultados: Las proyecciones muestran que Puerto Rico puede tener una TGF de 1.1 acotada por un intervalo de credibilidad del 95% (0.56,1.77) para el 2050. Conclusión: Bajo este escenario, Puerto Rico puede tener una disminución demográfica mayor que la proyectada por la Oficina del Censo y las Naciones Unidas.

References

1. Raftery AE, Li N, Sevcikova H, Gerland P, Heilig GK. Bayesian probabilistic population projections for all countries. Proceedings of the National Academy of Sciences. 2012;109(35):13915-13921. doi:https://doi.org/10.1073/pnas.1211452109
2. Alkema L, Raftery AE, Gerland P, et al. Probabilistic Projections of the Total Fertility Rate for All Countries. Demography. 2011;48(3):815-839. doi:https://doi.org/10.1007/s13524-011-0040-5
3. Alkema L, Gerland P, Raftery A, Wilmoth J. The United Nations Probabilistic Population Projections: An Introduction to Demographic Forecasting with Uncertainty. Foresight (Colche ster,Vt).2015;2015(37):19-24. https://pubmed.ncbi.nlm.nih.gov/26617476/.Accessed Febraury 25, 2024.
4. InternationalDatabase. https://www.census.gov/data-tools/demo/idb/#/dashboard?COUNTRY_YEAR=2024&COUNTRY_YR_ANIM=2024.Accessed Febraury 25,2024.
5. United Nations. World Population Prospects - Population Division - United Nations. un.org. Published 2022. https://population.un.org/wpp/. Accessed Febraury 25, 2024.
6. Abel, J. R., & Deitz, R. (2014). The causes and consequences of Puerto Rico's declining population. Current issues in Economics and Finance, 20(4). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2477891. Accessed May 8, 2023.
7. Hinojosa, J., Meléndez, E., & Pietri, K. S. (2019). Population decline and school closure in Puerto Rico. New York: Center for Puerto Rican Studies..https://www.luminafoundation.org/resource/population-decline-and-school-closure-in-puerto-rico/. Accessed February 25, 2024
8. Cohn D, Patten E, Mark Hugo Lopez. Puerto Rican Population Declines on Island, Grows on U.S. Mainland. Policycommons.net. Published August 11, 2014. https://policycommons.net/artifacts/619700/puerto-rican-population-declines-on-island-grows-on-us/1600874/. Accessed May 8, 2023
9. Liu P, Raftery AE, Ševcıková H. Probabilistic Estimation and Projection of the Annual Total Fertility Rate Accounting for Past Uncertainty: A Major Update of the bayesTFR R Package. arXiv:220706593

[stat]. Published online July 13, 2022.. https://arxiv.org/abs/2207.06593. Accessed May 8, 2023
10. Gietel-Basten, S., Sobotka, T., Zeman, K., Lutz, W., Butz, W. P., & KC, S. (2014). Future fertility in low fertility countries. World population and human capital in the twenty-first century: An overview, 39-146.
11. Anderson, Barbara A. “Projecting low fertility: Some thoughts about the plausibility and implications of assumptions.” Demographic Review 1, no. 1 (2014): 57.
12. Gavin W. Jones (2019) Ultra-low fertility in East Asia: policy responses and challenges, Asian Population Studies, 15:2, 131-149, DOI: 10.1080/17441730.2019.1594656
13. Wolfgang Lutz, Vegard Skirbekk and Maria Rita Testa. 2006. “The Low-Fertility Trap Hypothesis: Forces that May Lead to Further Postponement and Fewer Births in Europe,” Vienna Yearbook of Population Research, 4: 167-192
14. Ševcıková H, Alkema L, Raftery A. bayesTFR: An R Package for Probabilistic Projections of the Total FertilityRate. Journalof Statistical Software. 2011;43(1). doi:https://doi.org/10.18637/jss.v043.i01
15. Warren CW. Fertility determinants in Puerto Rico. Studies in Family Planning. 1987;18(1):42-48. Accessed May 8, 2023. https://pubmed.ncbi.nlm.nih.gov/3824423/